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General expressions for VM dominated form factors of hadrons fulfilling asymptotic conditions

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Abstract. General expressions for vector-meson dominated (VMD) form factors of hadrons fulfilling asymptotic conditions, derived previously for n vector-meson parameterization of the electromagnetic form factor of any strongly interacting particle with the asymptotics $\sim_{|t|\to\infty} t^{-m}$ $(m < n)$ and form factor normalization conditions, are presented. The special case of $m = n$ and the expression fulfilling asymptotic conditions without any form factor normalization are discussed too.

1 Introduction

Recently, starting with different properties of the electromagnetic (EM) form factor (FF) $F_h(t)$ of a strongly interacting particle to be saturated by n vector mesons and possessing the asymptotic behavior $\sim_{|t|\to\infty} t^{-m}$ $(m \leq n)$, two dissimilar systems of $(m-1)$ linear homogeneous algebraic equations for coupling constant ratios of vector mesons to the hadron under consideration were derived [1]. Though they really look differently, in [1] it has been demonstrated explicitly that both systems are exactly equivalent.

In this paper we are concerned with a more simple one, derived by means of the superconvergent sum rules for the imaginary part of the EM FF, in which the coefficients are simply even powers of the corresponding vector-meson masses. In more detail, we look for general expressions of VMD form factors $F_h(t)$ with the required asymptotics.

There are three cases appearing in various physical situations that are interesting, and all of them are discussed in this paper.

The first one appears in the construction of the unitary and analytic model of EM structure [2] of any strongly interacting particle with a number of building quarks $n_q > 2$, when at the first stage one has need for the VMD parameterization of the FF under consideration with the required asymptotics and normalization. The latter is found by a combination of the $(m-1)$ asymptotic conditions with the FF normalization condition and by a general solution of the obtained m linear algebraic equations for n coupling constant ratios. As a result, the FF depends then on the $(n-m)$ coupling constant ratios as free parameters of the model.

The second case is obtained from the previous one for $m \equiv n$ and it leads to expressions of all coupling constant ratios through the vector-meson masses. If the latter are known, numerical values of the coupling constant ratios are found, like in [3], for tensor coupling constants of vector mesons to nucleons.

The third case appears naturally in the determination of the behavior of the strangeness FF of strongly interacting particles from the isoscalar parts of the corresponding EM FFs. For instance, the value of the strangeness nucleon magnetic moment μ_s is unknown in advance and, thus, the corresponding strangeness magnetic FF (as a consequence also the strangeness Pauli FF) model is constructed without the normalization [4]. In order to keep some inner analytic structure of the corresponding EM form factor model, one has to construct it also without any normalization, though in the electromagnetic case it is exactly known experimentally to be equal to the magnetic moment of the nucleon. So in such a situation one has to solve the asymptotic conditions in the form of $(m-1)$ linear homogeneous algebraic equations for n coupling constant ratios. The resultant solutions express the $(m-1)$ coupling constant ratios through the other $(n-m+1)$ ones which are then free parameters of the model.

More details of the general solutions of asymptotic conditions and their consequences for all three specific cases can be found in the next section. The last section is devoted to conclusions and a discussion.

2 General solution of asymptotic conditions

First, we look for a general solution of the asymptotic conditions to be combined with the FF norm when the FF

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is saturated by more vector-meson resonances than the power determining the FF asymptotics.

If we assume that the EM FF of any strongly interacting particle is well approximated by a finite number n of vector-meson exchange tree Feynman diagrams, one finds the VMD pole parameterization

$$F_h(t) = \sum_{i=1}^{n} \frac{m_i^2}{m_i^2 - t}(f_{ihh}/f_i), \quad (1)$$

where $t = -Q^2$ is the momentum transfer squared of the virtual photon, m_i are the masses of vector mesons, and f_{ihh} and f_i are the coupling constants of the vector meson to the hadron and the vector-meson–photon transition, respectively. Furthermore, let us assume that the EM FF in (1) has the asymptotic behavior

$$F_h(t)_{|t|\to\infty} \sim t^{-m}, \quad (2)$$

and it is normalized at $t = 0$ as follows:

$$F_h(0) = F_0. \quad (3)$$

The requirement for the conditions (3) and (2) to be satisfied by (1) (including also the results of [1]) leads to the following system of m linear algebraic equations:

$$\sum_{i=1}^{n} a_i = F_0, \quad (4)$$
$$\sum_{i=1}^{n} m_i^{2r} a_i = 0, \quad r = 1, 2, \ldots, m-1,$$

for the n coupling constant ratios $a_i = (f_{ihh}/f_i)$. Therefore, a solution of (4) is looked for where the m unknowns $a_1, \ldots, a_m$ and $a_{m+1}, \ldots, a_n$ are considered as free parameters of the model. Then the system (4) can be rewritten in the matrix form

$$\mathbf{M}\mathbf{a} = \mathbf{b}, \quad (5)$$

with the $m \times m$ Vandermonde matrix $\mathbf{M}$

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & \ldots & 1 \\ m_1^2 & m_2^2 & \ldots & m_m^2 \\ m_1^4 & m_2^4 & \ldots & m_m^4 \\ \ldots & \ldots & \ldots & \ldots \\ m_1^{2(m-1)} & m_2^{2(m-1)} & \ldots & m_m^{2(m-1)} \end{pmatrix} \quad (6)$$

and the column vectors

$$\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_m \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} F_0 - \sum_{k=m+1}^{n} a_k \\ -\sum_{k=m+1}^{n} m_k^2 a_k \\ -\sum_{k=m+1}^{n} m_k^4 a_k \\ \ldots \\ -\sum_{k=m+1}^{n} m_k^{2(m-1)} a_k \end{pmatrix}. \quad (7)$$

Since the Vandermonde determinant of the matrix (6) is different from zero,

$$\det\mathbf{M} = \prod_{\substack{j,l=1, \\ j<l}}^{m} (m_l^2 - m_j^2). \quad (8)$$

by means of Cramer's rule a non-trivial solution of (5) (for more details see [5]) is obtained. We have

$$a_i = \frac{F_0(-1)^{1+i} \prod_{\substack{j=1\\ j\neq i}}^{m} m_j^2 \prod_{\substack{j,l=1\\ j<l, j,l\neq i}}^{m} (m_l^2 - m_j^2)}{\prod_{\substack{j,l=1\\ j<l}}^{m} (m_l^2 - m_j^2)} \quad (9)$$
$$- \frac{(-1)^{i-1} \prod_{\substack{j,l=1\\ j<l, j,l\neq i}}^{m} (m_l^2 - m_j^2) \sum_{k=m+1}^{n} a_k \prod_{\substack{j=1\\ j\neq i}}^{m} (m_j^2 - m_k^2)}{\prod_{\substack{j,l=1\\ j<l}}^{m} (m_l^2 - m_j^2)}.$$

giving the form factor $F_h(t)$ to be saturated by n-vector mesons ($n > m$) in the form suitable for the unitarization

$$F_h(t) = F_0 \frac{\prod_{j=1}^{m} m_j^2}{\prod_{j=1}^{m} (m_j^2 - t)} \quad (10)$$
$$+ \sum_{k=m+1}^{n} \left\{ \sum_{i=1}^{m} \frac{m_k^2}{(m_k^2 - t)} \frac{\prod_{\substack{j=1\\ j\neq i}}^{m} m_j^2}{\prod_{\substack{j=1\\ j\neq i}}^{m} (m_j^2 - t)} \frac{\prod_{\substack{j=1\\ j\neq i}}^{m} (m_j^2 - m_k^2)}{\prod_{\substack{j=1\\ j\neq i}}^{m} (m_j^2 - m_i^2)} - \frac{\prod_{j=1}^{m} m_j^2}{\prod_{j=1}^{m} (m_j^2 - t)} \right\} a_k,$$

for which the asymptotic behavior (2) and the normalization (3) are fulfilled automatically.

Now we consider the case of (4) for $n = m$. Then it can also be rewritten into the matrix form (5) with the $m \times m$ Vandermonde matrix (6) and the same column vector $\mathbf{a}$, but with the $\mathbf{b}$ vector of the following form:

$$\mathbf{b} = \begin{pmatrix} F_0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}. \quad (11)$$

The corresponding solutions

$$a_i = F_0 \frac{(-1)^{1+i} \prod_{\substack{j=1\\ j\neq i}}^{m} m_j^2 \prod_{\substack{j,l=1\\ j<l, j,l\neq i}}^{m} (m_l^2 - m_j^2)}{\prod_{\substack{j,l=1\\ j<l}}^{m} (m_l^2 - m_j^2)} \quad (12)$$
$$= F_0 \frac{\prod_{\substack{j=1\\ j\neq i}}^{m} m_j^2 (-1)^{1+i}}{\prod_{\substack{j=1\\ j\neq i}}^{m} (m_j^2 - m_i^2)(-1)^{i-1}}$$

are again found by means of Cramer's rule, and they are completely expressed only through the masses of m vector mesons, by means of which the considered FF is saturated.

The third case with the $(m-1)$ linear homogeneous algebraic equations for n ($n > m$) coupling constant ratios without any normalization of the FF appears naturally e.g. in the determination of the behavior of the strangeness FFs of strongly interacting particles from the isoscalar parts of the corresponding EM FFs, as we have mentioned in the Introduction.

Then, we have only the equations

$$\sum_{i=1}^{n} m_i^{2r} a_i = 0, \quad r = 1, 2, \dots m-1, \tag{13}$$

which can be rewritten in the matrix form (5) with the $(m-1) \times (m-1)$ matrix $\mathbf{M}$

$$\mathbf{M} = \begin{pmatrix} m_1^2 & m_2^2 & \dots & m_{m-1}^2 \\ m_1^4 & m_2^4 & \dots & m_{m-1}^4 \\ \dots & \dots & \dots & \dots \\ m_1^{2(m-1)} & m_2^{2(m-1)} & \dots & m_{m-1}^{2(m-1)} \end{pmatrix} \tag{14}$$

and the column vectors

$$\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{m-1} \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} -\sum_{k=m}^{n} m_k^2 a_k \\ -\sum_{k=m}^{n} m_k^4 a_k \\ -\sum_{k=m}^{n} m_k^6 a_k \\ \dots \\ -\sum_{k=m}^{n} m_k^{2(m-1)} a_k \end{pmatrix}. \tag{15}$$

The determinant of the matrix (14)

$$\det\mathbf{M} = \prod_{j=1}^{m-1} m_j^2 \prod_{\substack{j,l=1 \\ j<l}}^{m-1} (m_l^2 - m_j^2) \tag{16}$$

is again different from zero, and so a non-trivial solution of (13) of Cramer's rule exists in the form

$$a_i = -\sum_{k=m}^{n} \frac{m_k^2}{m_i^2} \frac{\prod_{j=1, j\neq i}^{m-1} (m_j^2 - m_k^2)}{\prod_{j=1, j\neq i}^{m-1} (m_j^2 - m_i^2)} a_k, \quad i = 1, 2 \dots, m-1, \tag{17}$$

giving the parameterization of the FF

$$F_h(t) = \sum_{k=m}^{n} \frac{\prod_{j=1}^{m-1} (m_j^2 - m_k^2)}{\prod_{j=1}^{m-1} m_j^2} \frac{\prod_{j=1}^{m-1} m_j^2}{\prod_{j=1}^{m-1} (m_j^2 - t)} \frac{m_k^2}{m_k^2 - t} a_k, \tag{18}$$

for which the asymptotic behavior (2) is fulfilled automatically.

3 Conclusions

General expressions for the VMD FFs of hadrons fulfilling asymptotic conditions, derived by means of the superconvergent sum rules for the imaginary part of the FF under consideration, in which coefficients are simply even powers of the corresponding vector-meson masses, have been found.

We have distinguished three cases appearing in various physical situations:
(i) in the construction of unitary and analytic models of the EM structure of any strongly interacting particle with a number of building quarks $n_q > 2$, when at the first stage one has need for the VMD parameterization of the FF to be saturated with n different vector mesons, but with the required asymptotics (2) and normalization (3), under the assumption $m < n$;
(ii) in the same problem; however, when $m = n$;
(iii) in a prediction of the behavior of the strangeness form factors of a strongly interacting particle from the isoscalar parts of the corresponding electromagnetic form factors.

In the first case, we have found the explicit form (10) of the EM FF for which the asymptotic behavior (2) and for $t = 0$ the normalization (3) are fulfilled automatically. Such a form is the starting point in a construction of the unitary and analytic model of EM structure of any strongly interacting particle, in which a superposition of complex conjugate pairs vector-meson poles on unphysical sheets of the four sheeted Riemann surface and continua contributions are considered at the same time.

In the second case, the explicit expressions (12) of all considered coupling constant ratios are found to be expressed through the masses of saturated vector mesons and the norm F_0 of the FF. The direct application of (12) to nucleons [6] gives a surprising coincidence with the values obtained in a fit [7] of the existing experimental data by the modified VMD model.

In the third case, the explicit form (18) of the isoscalar part of the EM FF of the strongly interacting particle was obtained, by means of which the behavior of the strangeness magnetic FF can be predicted.

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